deflection in an impedance transformer would be limited to a few thousandths of an inch to prevent power leakage past the shorts and the stress in the guide walls would, consequently, be very small. On the other hand, a piece of interconnecting waveguide might be permitted to deflect to 5 per cent or more over its original dimensions, in which case the yield stress could well become the limiting factor. In passing, it should be noted that it is seldom necessary to calculate the change in the broad dimension as this will be less than one-quarter of the narrow dimension change.

Where pressures are involved that are in excess of the capacity of the standard waveguide under question, a number of strengthening procedures can be employed. Most of the reinforcement methods can be expressed as an increase of the average wall thickness or section modulus, thus making possible the use of (1)–(4) to determine the required increase in section. Simplified formulas (5)–(7) can also be employed for many such cases by recalling that pressure-carrying capacity varies directly as the square of wall thickness and that deflection varies inversely as the cube of wall thickness.

Perhaps the most common means of strengthening waveguide is the addition of brazed, welded, or bolted braces, the design of which at minimum weight and cost involves considerable ingenuity. A second method is that of simply increasing the over-all waveguide wall thickness, utilizing such processes as precision cored sand casting or by welding plate together to form a waveguide; under such conditions, a useful but generally overlooked weight advantage can be obtained by making the narrow wall thinner than the broad wall, since, as noted in a previous paragraph, the change of the broad dimension under pressure is less than one-quarter that of the narrow dimension change. It is also possible to construct waveguide that is both stronger and lighter than standard guide. Successful means of accomplishing this end are 1) reinforced plastic guide coated with conductive metallic resin, and 2) a waveguide wall construction composed of two thin sheets of aluminum sandwiched over an aluminum honeycomb core. This latter method has resulted in construction of L-band honeycomb waveguide weighing 60 per cent of standard aluminum guide and increasing strength by a factor of four; it also appears likely that if the weight of the honeycomb guide is made equal to standard waveguide, the strength ratio would be approximately twenty to one. Both the reinforced plastic and honeycomb waveguide have been built and the former has already been utilized in radar systems by the Microwave Electronics Division of the Sperry Gyroscope Company.

It is apparent that the wall thicknesses of standard waveguide were not established with regard to pressure carrying capacity. As shown in the last column of Table II, a wide variation exists in this respect for the different waveguide sizes. Specially drawn material is available[1] to carry high pressure in large size guide. It would seem worthwhile to carry this approach a step further and provide reduced wall thicknesses in the smaller sizes to obtain lighter weight, particularly for airborne applications wherein pressures are not too great. With reasoning of this type as a basis, it appears that an investigation of a revised standardization for waveguide wall thicknesses would be justified.

The Calibration of Microwave Attenuators by an Absolute Method*

ELIZABETH LAVERICK†

Summary—**A bridge method by means of which microwave attenuators can be calibrated absolutely is described, with a consideration of the main possible sources of error. A bridge was set up at $\lambda_0 = 3.2$ cm to test the principle of the method. It was shown that, using nonspecialized equipment, a high degree of accuracy was obtainable. An attenuator was calibrated over a range of 20 db, with an accuracy of the order of ± 0.02 db. This accuracy is within the accuracy of other methods of calibration in current use, and there seems no reason why, with suitable precautions, the order of accuracy should not be improved still further, if required.**

* Manuscript received by the PGMTT, March 26, 1957; revised manuscript received, June 26, 1957.

† Microwave Division, Elliott Brothers (London) Ltd., Borehamwood, Herts., Eng.

Introduction

THE MAIN method in current use of calibrating microwave attenuators involves calibration in terms of a standard attenuator, usually the piston attenuator,[1–3] whose law of attenuation is accurately

[1] C. G. Montgomery (ed.), "Technique of Microwave Measurements," M.I.T. Rad. Lab. Ser. No. 11, McGraw-Hill Book Co., Inc., New York, N. Y., Ch. 11 and 13; 1947.

[2] L. G. H. Huxley, "A Survey of the Principles and Practice of Waveguides," Cambridge University Press, Cambridge, Eng., pp. 57–61; 1947.

[3] G. F. Gainsborough, "A method of calibrating standard signal generators and radio-frequency attenuators," *J. IEE*, pt. III, vol. 94, pp. 203–210; May, 1947.

calculable. As the frequency band is extended, manufacturing tolerances on the standard attenuator become tighter and the accuracy of the standard decreases rapidly. To overcome this, the usual technique is to compare the microwave attenuator with a standard attenuator which is operating at an intermediate frequency. By this means, the accuracy of that standard is maintained, and calibrations correct to ± 0.02 db are obtained. However, the auxiliary electronic equipment required is extensive and complex, and the method is neither direct nor absolute.

The following text describes a method whereby microwave attenuators may be calibrated absolutely and directly, at the frequency of operation. The accuracy obtainable is at least comparable with that of the standard attenuators in current use, and the method has the advantages that it is comparatively simple, and uses microwave and electronic apparatus which is generally available in most microwave laboratories.

Method of Calibration

The circuit consists essentially of a three-arm waveguide bridge or network as shown in Fig. 1, fed by a single oscillator. The bridge output is fed to a detector. Arm S contains the attenuator to be calibrated (X), a level-setting attenuator (L), and a phase shifter. Arm P contains a variable uncalibrated attenuator and a phase shifter. Arm Q contains a fixed attenuator such that it permits a signal of some suitable amplitude a and arbitrary phase to appear at the detector. Arms P and Q also contain waveguide switches. The procedure for calibration is as follows:

1) With arm P out of circuit and the unknown attenuator X at maximum attenuation, the level-setting attenuator L and phase shifter in arm S are adjusted for zero reading at the detector. When this occurs, the signal at the detector from arm S must be equal in amplitude and of opposite phase to that from arm Q; *i.e.*, the signal from S is $-a$.
2) With arm Q switched out and arm P switched in, the attenuator and phase shifter in arm P are adjusted for zero reading at the detector, the signal from P being $+a$.
3) With all three arms switched in, we have $+2a$ from P and Q and $-a$ from S. To obtain zero output, the unknown attenuator X in arm S must be adjusted to give $-2a$ at the detector; *i.e.*, four times the original power. X is consequently changed by -6.02 db. Unless the attenuator in arm S is phase shiftless it will be necessary to readjust the phase shifter in arm S for zero output. Repeating steps 2 and 3, S is repeatedly adjusted to give amplitudes $-3a$, $-4a$, $\cdots$, $-na$, the corresponding attenuation steps being -3.52 db, -2.50 db, etc.

The magnitude of the fixed attenuator and the number of steps $(n-1)$ are chosen according to the range of attenuation over which calibration is required. For example, if it is required to cover a range of 20 db, 9 steps are used $(n=10)$ and the fixed attenuator is chosen to be slightly greater than 20 db. It is not necessary, however, to know the exact magnitude of the fixed attenuator.

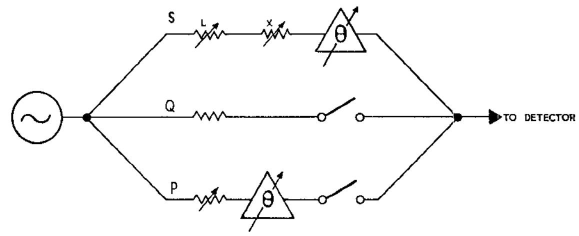

Fig. 1—Bridge network for the absolute calibration of attenuators.

It will be observed that the calibration is carried out in a direction of increasing amplitude or decreasing attenuation; *i.e.*, the starting point of the calibration is at the maximum attenuation end of the range, whereas it is usually required to calibrate an attenuator from the "zero" or minimum attenuation point. For instance, in the example quoted above, at the tenth step the attenuator is adjusted to -20 db with respect to the starting point. In general, however, X will not then be exactly at the "zero." In order to adjust for this, the attenuator is set at the "zero" and the balance condition reobtained by adjusting the level-setting attenuator in arm S. The calibrating procedure is then completely repeated, step 1 giving the point 20 db up on the zero and step 9 giving "zero," or minimum attenuation.

Apparatus

The Microwave System

The microwave bridge is shown in Fig. 2. The power is divided in a known ratio between the three arms of the bridge network by means of two hybrid-T junctions. Arm S contains a level-setting attenuator, a phase shifter, and the attenuator to be calibrated, X; Arm Q contains a switch (consisting of a highly attenuating vane which can be switched in or out of the waveguide as required) and the fixed attenuator; and Arm P contains a switch, phase shifter, and variable attenuator. The signals from the three arms are recombined by means of two further hybrid T's, and the bridge output taken to some form of detector.

The Detecting System

The source (a reflex-klystron) is square-wave modulated at 1 kc. In order to achieve the necessary sensitivity, a superheterodyne detecting system is employed and to this end a hybrid-T balanced mixer with reverse polarity crystals is used. The square-wave modulated IF signal from the mixer is amplified and passed to an audio-frequency amplifier, the output from which is indicated by a meter.

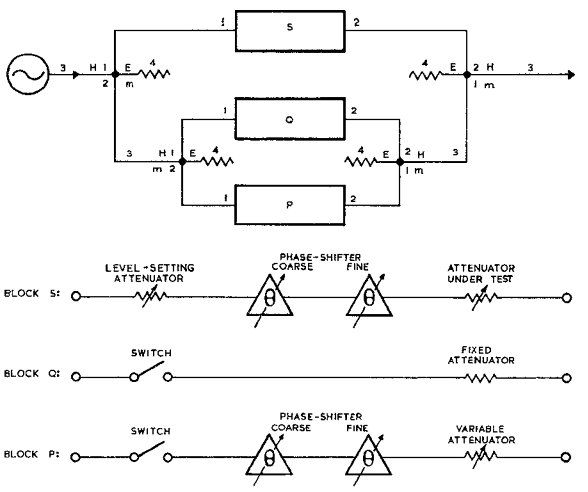

Fig. 2—Circuit for studying the effects of multiple reflections within the bridge network.

Sources of Error

Multiple Reflections Within the Bridge Network

Because of multiple reflections, cross coupling will occur between the arms of the bridge network and this will result in an error of measurement. An estimate of the magnitude of this error can be made from a knowledge of the reflection and transmission coefficients of the bridge arms. For example, it can be shown that for a calibration over a 20-db range the total error δ is given by

$$\delta = 20 \log \left(\frac{10 + \Delta}{10}\right) \text{db}$$

where, in terms of the moduli of the scattering coefficients,

$$\Delta \leq 10\Delta s_{11}(m_{11} + m_{12}) + 10\Delta s_{22}(m_{12} + m_{22})$$
$$+ \frac{m_{22} + 3m_{12}}{2}\left[45(q_{11} + q_{11}^{x}) + 9(p_{11} + p_{11}^{x})\right]$$
$$+ \frac{m_{11} + 3m_{12}}{2}\left[45(q_{22} + q_{22}^{x}) + 9(p_{22} + p_{22}^{x})\right]. \quad (1)$$

In this equation

m = hybrid T's,
s = arm S,
q = arm Q in circuit,
q^x = arm Q when switched out of circuit,
p = arm P in circuit,
p^x = arm P when switched out of circuit (see Fig. 2),
Δs_{11} and Δs_{22} = change in S_{11} and S_{22}, respectively, during the measurement procedure.

This result is based on the following assumptions:

1) Equal power-split in the hybrid T's.
2) Reflections are small, therefore terms involving more than two reflections can be neglected.
3) The attenuation in each of the three arms in always such that terms in p_{12}^3, q_{12}^3, s_{12}^3 can be neglected.

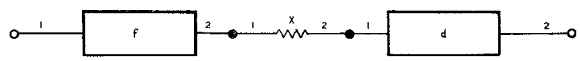

Fig. 3—Block S in Fig. 2. Circuit for studying the effects of multiple reflections within the arm of the bridge network containing the unknown attenuator, X.

4) Measurements are made only when the bridge is balanced, and effectively zero signal is incident on the detector. Under these circumstances the mismatch of the detector can be neglected.
5) Since s, p, and p^x vary during the course of the calibration, maximum values have been assumed in order to estimate the maximum possible error.

By placing suitable attenuating pads at both ends of each arm, and thereby reducing the effect of reflections, the error in calibration can be decreased accordingly.

It is clear from the form of (1) that this error increases with the range of attenuation.

Multiple Reflections Within the Arm of the Bridge Network Containing the Unknown Attenuator X

It is essential that multiple reflections within the arm of the bridge network containing the attenuator under test should be kept to within certain limits depending on the accuracy required.[4] This is achieved by placing attenuating pads on both sides of the attenuator under test, which is the normal condition of operation for any attenuator when high accuracy is required.

It can be shown that the error in calibration arising from multiple reflections within the arm containing the attenuator is given by

δ, in db where

$$\delta = 20 \log_{10}\left(\frac{10 + \Delta}{10}\right),$$

$$\Delta \leq 2f_{22}X_{11} + 2d_{11}X_{22} + 2f_{22}d_{11}X_{12}^2, \text{ and} \quad (2)$$

X, f and d are moduli of the scattering coefficients. X refers to the attenuator under test, and f and d refer to the match on either side of the attenuator under test (see Fig. 2 and Fig. 3).

Leakage through Switches

A third source of error arises from the finite transmission coefficient of the open-circuited bridge arms. In the case of a 20-db calibration range it can be shown that the error arising from this is

$$\delta = 20 \log \left(\frac{10 + \Delta}{10}\right) \text{db}$$

where

$$\Delta \leq \frac{18\delta_q^{x}}{q_{12}} = 180\delta_q^{x} \quad (3)$$

[4] R. W. Beatty, "Mismatch errors in the measurement of μ.h.f. and microwave variable attenuators," *J. Res. NBS*, vol. 52, pp. 7–9; January, 1954.

and δ_q^x = modulus of transmission coefficient through the switch in arm Q (see Fig. 2).

It is assumed above that the switches in arms P and Q are identical.

Experimental Results

In order to test this method of calibrating attenuators, a bridge was set up at a wavelength of 3.2 cm, and a rotary attenuator was calibrated over a 20-db range. A detailed circuit of the bridge network is shown in Fig 4.

Performance of the Individual Components of the Bridge Network Hybrid T's

At $\lambda_0 = 3.2$ cm the moduli of the scattering coefficients are as follows:

$$|m_{11}| = 0.033$$
$$|m_{22}| = 0.028$$
$$|m_{12}| = 0.040.$$

Attenuators

Rotary attenuators were used as the fixed and variable attenuators in arms P and Q. Their reflection coefficients were ≤ 0.05. The level-setting attenuator L, in arm S, was of the guillotine type using a resistive vane. Its reflection coefficient was ≤ 0.02.

Switches

Each switch consisted of a length of waveguide into which a highly attenuating vane could be inserted through a slot in the broad face. The mechanism was completely shielded to eliminate radiation. The switches had a reflection coefficient of 0.026 and a transmission coefficient <0.0001 with the vane fully in; and effectively zero reflection coefficient and unity transmission coefficient with the vane fully out.

Phase Shifters

The coarse phase shifters which were used to roughly equalize the path lengths of the bridge arms were of the "trombone" type having a reflection coefficient of 0.06, which is constant with change of phase. The fine phase shifters, used to obtain the final phase balance, were shielded slotted squeeze sections with a reflection coefficient of 0.006.

Attenuating Pads

The attenuating pads used to minimize the effects of multiple reflections were made up of tapered resistive vanes, of reflection coefficient ≤ 0.005.

Estimate of Padding Necessary to Attain Required Accuracy. Multiple Reflections Within the Bridge Network

Referring to (1) and Fig. 2, and substituting the practical values given above for the various components,

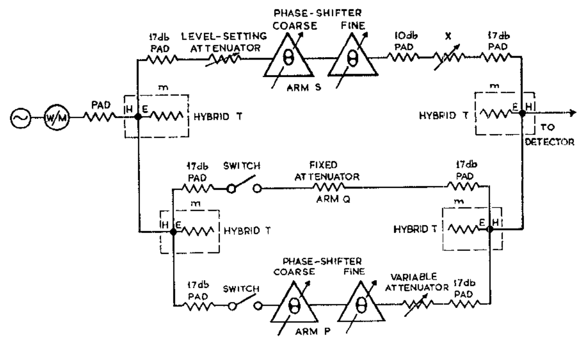

Fig. 4—Detailed circuit of the bridge network.

$$\Delta s_{22} = \Delta s_{11} \leq 2s_{11} \leq 2(0.02 + 0.066 + 0.05) = 0.272$$
$$q_{11} = 0.052 \qquad q_{22} = 0.052$$
$$q_{11}^x = 0.026 \qquad q_{22}^x = 0.052$$

assuming 20 db in the fixed attenuator; *i.e.*, $q_{12} = 0.1$.

$$p_{11} = (0.066 + 0.05) = 0.116$$
$$p_{11}^x = 0.026$$
$$p_{22}^x = (0.05 + 0.001) = 0.051$$
$$p_{22}^x = (0.05 + 0.001) = 0.051;$$
$$\therefore \Delta \leq 2.72(0.033 + 0.08 + 0.028) + 0.074(3.51 + 1.278) + 0.077(4\ 68 + 0.918) = 1.169.$$

Using 17-db pads at both ends of P, Q, and S, Δ is reduced to ≤ 0.025, or the error over 20 db is ≤ 0.021.

Multiple Reflections Within the Arm Containing the Unknown Attenuator X

Referring to (2) and Fig. 2 and Fig. 3, it can be seen that

$$f_{22} = 0.062 + 0.02 + 0.033 = 0.115$$
$$d_{11} = 0.028$$
$$X_{11} = X_{22} = 0\ 05;$$
$$\therefore \Delta \leq (0.23 \times 0.05) + (0.056 \times 0.05) + (0.23 \times 0.028) = 0.021 \text{ or } \delta \leq 0.02 \text{ db}.$$

In practice a 17-db pad is inserted between the unknown attenuator X and the hybrid T, as a result of the consideration of multiple reflections in the bridge network. By inserting a 10-db pad between X and the phase shifters, the error calculated above is reduced to a negligible amount.

Leakage through Switches

Referring to (3) and substituting $\delta_q^x = 0.0001$,

$$\Delta \leq 0.018$$

and

$$\delta \leq 0.009 \text{ db}.$$

Result of Calibration

It is reasonable to suppose, on the basis of the above estimates, that the calibration of the rotary attenuator by this method should be accurate to ± 0.02 db over a 20-db range, the accuracy being greatest over the first part of the range.

The results obtained are shown in Fig. 5, where the error or deviation in db is plotted for various settings of the rotary attenuator, which is in itself an absolute instrument. For comparison, the same attenuator was calibrated against an IF piston attenuator, the results being shown in the same illustration. The deviations are, in both cases, of the order expected.

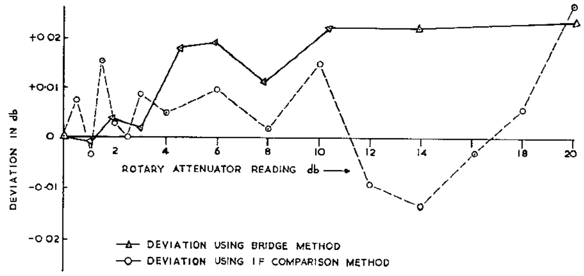

Fig. 5—Deviations of attenuation from rotary attenuator reading using the bridge method and IF comparison method of calibration. The rotary attenuator is an absolute instrument.

Conclusion

The method described above for the absolute calibration of microwave attenuators has been tested experimentally at a wavelength of 3.2 cm. Estimates were made of the padding required to attain an accuracy of the order of ± 0.02 db over a 20-db range. It was shown that the results obtained are accurate to within the design limits of the apparatus, and to within the accuracy of other methods in current use. By further reducing the effects of multiple reflections in the microwave circuit it should be possible to attain even greater accuracy, if required. The method is insensitive to power fluctuations and used comparatively simple and readily available microwave and electronic apparatus.

Acknowledgment

The author wishes to thank E. A. N. Whitehead for the basic idea, and her other associates of the Microwave Division of Elliott Brothers (London) Limited who helped to put the idea into practice. The comparison with a piston attenuator was carried out at the Radar Research Establishment at Malvern, Worcestershire, and the author would like to acknowledge their cooperation.

Pulse Waveform Degradation Due to Dispersion in Waveguide*

ROBERT S. ELLIOTT†

Summary—**Phase velocity in a waveguide is a nonlinear function of frequency and thus causes dispersion of the spectral components in a pulse waveform. For most practical cases, it is a good assumption to consider the phase constant to be a quadratic function of frequency. An expression can then be derived for the exit waveform shape as a function of guide length, dispersion, and width of the input rectangular pulse. The derived expression is given in terms of tabulated error functions and Fresnel integrals. It is universal in form and applicable to a wide range of practical problems. A family of degraded wave shapes has been computed from this expression and is presented graphically. The results apply for any mode in a straight waveguide of arbitrary but constant cross section.**

Introduction

As THE usable range of microwave frequencies has been pushed higher and higher, a run of waveguide whose physical length is L has assumed an electrical length great enough to affect transmission of pulsed energy, even when loss is ignored. The reason for this lies in the frequency behavior of the phase constant. If $\beta(\omega)$ is the phase constant, then

$$\beta(\omega) = \frac{2\pi}{\lambda_g} = \frac{\sqrt{\omega^2 - \omega_c^2}}{v} \tag{1}$$

in which λ_g is the guide wavelength at the angular frequency ω, and $v = (\mu\epsilon)^{-1/2}$, with μ and ϵ the permeability and permittivity, respectively, of the medium filling the guide. Eq. (1) applies for any mode in a straight section of waveguide of any constant cross section. ω_c is the cutoff angular frequency of the particular mode being considered.

Eq. (1) can be expanded in a Taylor's series about the angular frequency ω_0, giving

$$\beta(\omega) = \beta_0 + \frac{\omega_0}{v^2\beta_0}[\omega - \omega_0] - \frac{1}{2}\frac{\omega_c^2}{v^4\beta_0^3}[\omega - \omega_0]^2 + \frac{1}{2}\frac{\omega_0\omega_c^2}{v^6\beta_0^5}[\omega - \omega_0]^3 - \cdots \tag{2}$$

* Manuscript received by the PGMTT, April 4, 1957; revised manuscript received, June 14, 1957. The research reported in this paper was performed for the Allan B. DuMont Labs. under contract with the Naval Bur. of Aeronautics.

† RANTEC Corp., Calabasas, Calif.